Exercises A : Architecture and Calculus

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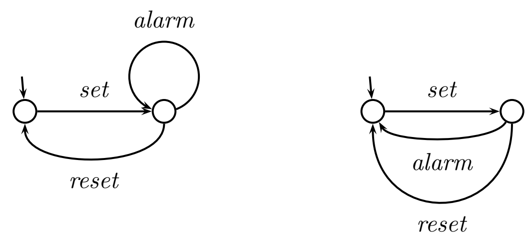
Exercise I.1

Consider two labelled transition systems representing two alternative behaviours of an alarm clock, as depicted below:



1. Describe each behaviour and distinguish between the two alarm clocks.
2. Describe one of these graphical specifications in the form of a labelled transition system conforming to the formal definition.
3. Add to alarm clock a new feature for specifying the number of alarms to go off.
4. Modify the previous specification to express a situation in which it is unclear how often the alarm can be repeated.
5. Draw the behaviour of an alarm clock where it is always possible to do a set or a reset action.
6. Draw the behaviour of an alarm clock with unreliable buttons. When pressing the set button the alarm clock can be set, but this does not need to be the case. Similarly for the reset button. Pressing it can reset the alarm clock, but the clock can also stay in a state where an alarm is still possible.
7. Draw the behaviour of an alarm clock where the alarm sounds at most three times when no other action interferes.

Exercise I.2

1. Define a binary operator $|||$ which models the parallel execution of its two arguments. Its transitions come from the weaving, or merging, of the transitions of its arguments. It is assumed that there is no interference between them.
2. Discuss, starting with an example, whether this operator is associative.
3. Discuss, starting with an example, whether this operator is commutative.

Exercise I.3

Suppose a labelled transition system is given by the following transition relation:

$$\{\langle 1,a,2\rangle, \langle 1,a,3\rangle, \langle 2,a,3\rangle, \langle 2,b,1\rangle, \langle 3,a,3\rangle, \langle 3,b,1\rangle, \langle 4,a,5\rangle, \langle 5,a,5\rangle, \langle 5,b,6\rangle, \langle 6,a,5\rangle, \langle 7,a,8\rangle, \langle 8,a,8\rangle, \langle 8,b,7\rangle\}$$

Prove or refute $1 \sim 4 \sim 6 \sim 7$.

Exercise I.4

Suppose that the existential quantifiers in the definition of bisimulation were replaced by universal quantifiers. Characterise the resulting bisimilarity relation.

Exercise I.5

Show that bisimilarity is strictly included in equisimilarity, and that the latter is also strictly included on trace equivalence.

Exercise I.6

Discuss whether bisimilarity $\sim$

- is closed for union
- is closed for intersection

Exercise I.7

Let $A(a) \triangleq a.A$ and $B(b) \triangleq \overline{b}.B$. Compute the first derivatives of the following processes:

1. $A + B$
2. $A + B\langle a\rangle$
3. $A \mid B$
4. $A \mid B\langle a\rangle$
5. $\{a/b\}\,(A \mid B)$
6. $\mathsf{new}\,\{a\}\;(A \mid B\langle a\rangle)$

Exercise I.8

Let $A(a, b, c, d) \triangleq \overline{a}.b.A + \overline{c}.d.A$. Draw the transition graphs of the following processes

1. A
2. $\mathsf{new}\,\{a\}\;A$

Exercise I.9

Consider the following description of a two-position *buffer* with acknowledgements. Note the process is built from copies of a 1-position *buffer* also with acknowledgements: it acknowledges in $\overline{r}$ the reception of a message and waits in t the confirmation that a message sent was arrived to its destination.

$$Bs \triangleq= \mathsf{new}\,\{mo, mi\}\;(B(in, mo, mi, r) \mid B(mo, out, t, mi))$$
$$B(in, out, t, r) \triangleq in.\overline{out}.t.\overline{r}.B$$

1. Draw the synchronisation graph of Bs.
2. Check whether the behaviour of Bs is the intended one (drawing, for this purpose, the corresponding transition graph)
3. Find a solution to the problem detected (if any) and draw the corresponding transition graph.
4. Explain how the specification given (or your new solution) can be adapted to describe *buffers* with an arbitrary, but fixed number of positions.

Exercise I.10

Consider the following description of a 1-position bidirectional buffer, *i.e.*, able to transmit and receive messages in any direction.

$$BT(in_1, in_2, out_1, out_2) \triangleq in_1(x).\overline{out_1}\langle x\rangle.BT + in_2(x).\overline{out_2}\langle x\rangle.BT$$

1. Specify a 2-position bidirectional buffer by parallel composition of two instances of process BT.
2. Draw its synchronisation diagram and the transition graph.

Exercise I.11

Consider the following specification of a control system for a crossing between a road and a railway. Events car and $train$ modelled, respectively, a car or a train approaching the cross. Actions up e dw stand for the opening and closing of the protection bar to prevent cars to cross. Similarly, $green$ and red model the semaphore for trains. Finally, events $\overline{ccross}$ and $\overline{tcross}$ come from sensors which register the actual cross of a car or a train, respectivelyy.

$$\begin{aligned} Road &\triangleq car.up.\overline{ccross}.\overline{dw}.Road \\ Rail &\triangleq train.green.\overline{tcross}.\overline{red}.Rail \\ Signal &\triangleq \overline{green}.red.Signal + \overline{up}.dw.Signal \end{aligned}$$

$$C \triangleq \mathsf{new}\,\{green, red, up, dw\}\ (Road \mid Rail \mid Signal)$$

1. Explain the behaviour of this process and sketch its synchronisation diagram.
2. Compute the transition graph corresponding to process C