Making Logics Coalgebraic

(PhD proposal)

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1 Motivation and Context

The formal specification of a non trivial software system often requires different logics to capture specific types of requirements or design issues: if properties of data structures are typically captured in an equational framework, behavioural issues will call for some sort of modal or temporal logic, whereas probabilistic reasoning will be required to predict or analyse faulty behaviour in distributed systems. This explains the growing interest in the systematic combination of logics, an area whose overall aim can be summed up in a simple methodological principle: distinguish the underlying nature of the requirements to be formalised, and then build a single logic for the whole system by combining whatever logics are suitable to handle the types of requirement found.

One way to achieve this is through what is called asymmetric combination of logics, a process that develops the characteristic features of a logic on top of another. Temporalisation [FG92], modalisation [DS07], and hybridisation [MMDB11] are instances of this sort of combination: they equip the base logic with suitable machinery to express and reason about transition structures, while at the same time maintaining the original logical features which are then used to enforce specific properties at each state. Actually, in the three cases the resulting logical models become transition structures of some type with each state corresponding to a model of the base logic.

At a generic level, the examples hint at an observational, coalgebraic semantics that abstracts their individual properties through two assumptions: one is that the transition type is given by coalgebras of some functor; the other is that each state (in the coalgebras) corresponds to a model of the logic being enriched. In fact, a very similar observation was already explored in a number of papers (*cf.* [KP11]), but in every case, to the best of the author's knowledge, the focus was on the different transition semantics, and not on the combination of the latter with an arbitrary logic. Another way to put this is that in the previous results the base logic was fixed. However, in order to accomplish the

methodological principle stated above, the base logic needs to vary according to whatever requirements the engineer encounters in the development process.

2 Objectives

The main goal of this PhD project is the development of a general notion of asymmetric combination of logic that turns logics coalgebraic. More concretely, a process that equips any given logic with suitable coalgebraic machinery respective to a given functor; this means that, in contrast to hybridisation, or temporalisation, the input parameter is not just the base logic, but also the functor that gives rise to the intended type of transitions.

A positive conclusion of the goal mentioned above yields a number of follow ups that should also be pursued during the PhD. One natural question concerns the identification of the logics that are possible to generate through the method; can we get new interesting combinations, not covered by temporalisation, modalisation, or hybridisation ? And which combinations are more relevant for software development processes ?

Another interesting line of work is to study the role of coalgebraic machinery, like (bi)simulation and finality, in this framework. For instance, is it possible for the logic being enriched to systematically 'import' the bisimulation methods that correspond to the functor given as input ? And are such methods suitable in the context of the resulting logic ?

Also worth studying is the generation of (sound and complete) calculi for the enriched logic from a calculus of the corresponding base logic and the given functor. Actually, this was already explored in the coalgebraic domain [KP11], where the base logic is fixed, but also for temporalisation [FG92], and hybridisation [NMB15]. The coalgebraic nature of the asymmetric combination process that this PhD project aims at, strongly suggests that most developments will take advantage of the knowledge and experience of both approaches.

References

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