Exercises 3 : Interaction and Concurrency

Luis Soares Barbosa

Exercise I.1

Let $A(a) \triangleq a.A$ and $B(b) \triangleq \overline{b}.B$. Compute the first derivatives of the following processes:

1. $A + B$
2. $A + B\langle a \rangle$
3. $A \mid B$
4. $A \mid B\langle a \rangle$
5. $(A \mid B)[a/b]$
6. $(A \mid B\langle a \rangle)\backslash_{\{a\}}$

Exercise I.2

Let $A(a, b, c, d) \triangleq \overline{a}.b.A + \overline{c}.d.A$. Draw the transition graphs of the following processes

1. A
2. $A\backslash_{\{a\}}$

Exercise I.3

Consider the following description of a two-position *buffer* with acknowledgements. Note the process is built from copies of a 1-position *buffer* also with acknowledgements: it acknowledges in $\overline{r}$ the reception of a message and waits in t the confirmation that a message sent was arrived to its destination.

$$Bs \triangleq= (B(in, mo, mi, r) \mid B(mo, out, t, mi))\backslash_{\{mo,mi\}}$$
$$B(in, out, t, r) \triangleq in.\overline{out}.t.\overline{r}.B$$

1. Draw the synchronisation graph of Bs.
2. Check whether the behaviour of Bs is the intended one (drawing, for this purpose, the corresponding transition graph)
3. Find a solution to the problem detected (if any) and draw the corresponding transition graph.
4. Explain how the specification given (or your new solution) can be adapted to describe *buffers* with an arbitrary, but fixed number of positions.

Exercise I.4

Consider the following description of a 1-position bidirectional buffer, *i.e.,*, able to transmit and receive messages in any direction.

$$BT(in_1, in_2, out_1, out_2) \;\triangleq\; in_1(x).\overline{out_1}\langle x\rangle.BT + in_2(x).\overline{out_2}\langle x\rangle.BT$$

1. Specify a 2-position bidirectional buffer by parallel composition of two instances of process BT.
2. Draw its synchronisation diagram and the transition graph.

Exercise I.5

Consider the following specification of a control system for a crossing between a road and a railway. Events car and $train$ modelled, respectively, a car or a train approaching the cross. Actions up e dw stand for the opening and closing of the protection bar to prevent cars to cross. Similarly, $green$ and red model the semaphore for trains. Finally, events $\overline{ccross}$ and $\overline{tcross}$ come from sensors which register the actual cross of a car or a train, respectivelyy.

$$\begin{aligned} Road &\triangleq car.up.\overline{ccross}.\overline{dw}.Road \\ Rail &\triangleq train.green.\overline{tcross}.\overline{red}.Rail \\ Signal &\triangleq \overline{green}.red.Signal + \overline{up}.dw.Signal \end{aligned}$$

$$C \triangleq (Road \mid Rail \mid Signal)\backslash_{\{green,red,up,dw\}}$$

1. Explain the behaviour of this process and sketch its synchronisation diagram.
2. Compute the transition graph corresponding to process C

Exercise I.6

An n-*trigger*, for $n > 1$, is used in electronic voting to detect that a fixed number of votes have been received along its n input ports, numbered from a_1 to a_n. As soon votes have been received in half of the input ports a signal is sent through its output port $\overline{s}$ and the process terminates. Each port a_i receives only a single input. Inputs, however, may arrive in any order to the different ports.

1. Specify a 3-*trigger*.
2. Specify a n-*trigger*, for n arbitrary.

Exercise I.7

Draw the transition graph of $T \triangleq a.(b.\mathbf{0} \mid T)$?