Exercises 1 : Interaction and Concurrency

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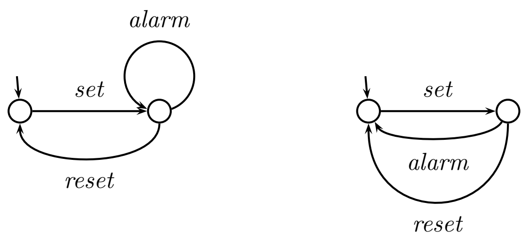
Exercise I.1

Consider two labelled transition systems representing two alternative behaviours of an alarm clock, as depicted below:



1. Describe each behaviour and distinguish between the two alarm clocks.
2. Describe one of these graphical specifications in the form of a labelled transition system conforming to the formal definition.
3. Add to alarm clock a new feature for specifying the number of alarms to go off.
4. Modify the previous specification to express a situation in which it is unclear how often the alarm can be repeated.
5. Draw the behaviour of an alarm clock where it is always possible to do a set or a reset action.
6. Draw the behaviour of an alarm clock with unreliable buttons. When pressing the set button the alarm clock can be set, but this does not need to be the case. Similarly for the reset button. Pressing it can reset the alarm clock, but the clock can also stay in a state where an alarm is still possible.
7. Draw the behaviour of an alarm clock where the alarm sounds at most three times when no other action interferes.

Exercise I.2

1. Define a binary operator $|||$ which models the parallel execution of its two arguments. Its transitions come from the weaving, or merging, of the transitions of its arguments. It is assumed that there is no interference between them.
2. Discuss, starting with an example, whether this operator is associative.
3. Discuss, starting with an example, whether this operator is commutative.

Exercise I.3

Consider the following two programs:

$$P_1 \triangleq \texttt{x := 2*x} \qquad \text{and} \qquad P_2 \triangleq \texttt{x := 1+x}$$

1. Draw, for each program, a transition system whose states represent the values variable `x` may take. Assume `x = 4` as the initial state in both cases.
2. Compute and draw $P_1 \mathbin{|||} P_2$.
3. Explain why, in the presence of interference (in this example through a shared variable), the interleaving operator may produce invalid states.

Exercise I.4

Consider the following alternative definition of a labelled transition system as a function

$$\mathsf{next} : S \times \mathcal{N} \longrightarrow \mathcal{P}\mathcal{S}$$

where S is the set of states. In this model a state transition $p \xrightarrow{a} p'$ is defined by

$$p \xrightarrow{a} p' \quad \equiv \quad p' \in \mathsf{next}(p, a)$$

A morphism between transition systems represented in this way is a function h such that the following diagram communtes

$$\begin{array}{ccc} S \times \mathcal{N} & \xrightarrow{\mathsf{next}} & \mathcal{P}S \\ {\scriptstyle h \times \mathsf{id}} \downarrow & & \downarrow {\scriptstyle \mathcal{P}h} \\ S' \times \mathcal{N} & \xrightarrow{\mathsf{next}'} & \mathcal{P}S' \end{array}$$

1. Express the definition of a morphism as an equation.
2. Discuss whether a morphism $h : \mathsf{next} \longrightarrow \mathsf{next}'$

   (a) *preseves* transitions:

   $$s' \in \mathsf{next}\ \langle s, a \rangle \Rightarrow h\ s' \in \mathsf{next}'\ \langle h\ s, a \rangle$$

   (b) *reflects* transitions:

   $$r' \in \mathsf{next}'\ \langle h\ s, a \rangle \Rightarrow \langle \exists\, s' \in S\ :\ s' \in \mathsf{next}\ \langle s, a \rangle\ :\ r' = h\ s' \rangle$$