Quantum Systems

(Lecture 6: The teleportation protocol)

Luís Soares Barbosa

Universidade do Minho

Teleportation

Aim: to transmit, using two classical bits, the state of a single qubit.

Surprisingly,

- shows that two classical bits suffice to communicate a qubit state, which has an infinite number of configurations
- provides a mechanism for the transmission of an unknown quantum state, in spite of the no-cloning theorem

Note that the original state cannot be preserved (precisely because of the no-cloning result), which motivates the name of the protocol ...

Teleportation

Strategy:

- Alice and Bob share a Bell state (also called a EPR pair)

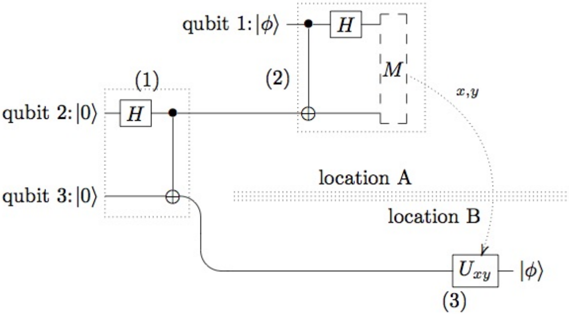
$$|r\rangle \;=\; \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

  created ahed in time when both qubits were together and can be made to interact to produce an entangled state (through e.g. a *CNOT* gate).

- This entangled state becomes a resource which remains available when one qubit is taken away from the other.
- Then Alice encodes the unknown qubit with her part of the EPR pair, measures and transmits the (classical) result of the measurement.
- Finally, Bob decodes its part of the EPR pair based on the (classical) information received

Teleportation

Implementation:



The EPR pair

is the real resource

named after Einstein, Podolsky, and Rosen, from the *hidden-variable* controversy

Teleportation

Alice

... has a qubit whose state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ she does not know, but wants to send to Bob through classical channels.

The starting point is the 3-qubit state after stage (1) whose first 2 qubits are controlled by Alice and the last by Bob:

$$\begin{aligned} |\phi\rangle \otimes |r\rangle &= \frac{1}{\sqrt{2}}(\alpha|0\rangle \otimes \overbrace{(|00\rangle + |11\rangle)}^{\text{entangled}} + \beta|1\rangle \otimes \overbrace{(|00\rangle + |11\rangle)}^{\text{entangled}}) \\ &= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle) \end{aligned}$$

Teleportation

Alice

... then she applies $CNOT \otimes I$ and $H \otimes I \otimes I$ to obtain

$$
\begin{aligned}
&(H \otimes I \otimes I)(CNOT \otimes I)(|\phi\rangle \otimes |r\rangle) \\
&= (H \otimes I \otimes I)\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle) \\
&= \frac{1}{2}(\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|010\rangle + |001\rangle - |110\rangle - |101\rangle)) \\
&= \frac{1}{2}(|00\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) + |01\rangle \otimes (\alpha|1\rangle + \beta|0\rangle) + \\
&\qquad + |10\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) + |11\rangle \otimes (\alpha|1\rangle - \beta|0\rangle))
\end{aligned}
$$

Teleportation

Alice

Alice measures the first two qubits and obtains one of the four standard basis states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, with equal probability.
Depending on the result of her measurement, the state of Bob's qubit is projected to

$$\alpha|0\rangle + \beta|1\rangle,\ \alpha|1\rangle + \beta|0\rangle,\ \alpha|0\rangle - \beta|1\rangle,\ \alpha|1\rangle - \beta|0\rangle$$

Then, Alice sends the result of her measurement as two classical bits to Bob.

After these transformations, crucial information about the original state $|\phi\rangle$ is contained in Bob's qubit, Alice's being destroyed ...

Teleportation

Bob

When Bob receives the two bits from Alice, he knows how the state of his half of the entangled pair compares to the original state of Alice's qubit.

Bob can reconstruct the original state of Alice's qubit, $|\phi\rangle$, by applying the appropriate decoding transformation to his qubit, originally part of the entangled pair.

Bits received	Bob's state	Transformation to decode
00	$\alpha|0\rangle + \beta|1\rangle$	I
01	$\alpha|1\rangle + \beta|0\rangle$	X
10	$\alpha|0\rangle - \beta|1\rangle$	Z
11	$\alpha|1\rangle - \beta|1\rangle$	Y

After decoding, Bob's qubit will be in the state $|\phi\rangle$

Superdense coding

Aim: encode and transmit two classical bits with one qubit and a shared EPR pair.

This result is surprising, since only one bit can be extracted from a qubit

The idea is that, since entangled states can be distributed ahead of time, only one qubit needs to be physically transmited to communicate two bits of information.

Let Alice (Bob) be sent and operate the first (second) qubit of the EPR pair

$$|r\rangle \;=\; \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Superdense coding

Alice

wishes to transmit the state of two classical bits encoding one of the numbers 0 through 3. Depending on this number, Alice performs one of the Pauli transformations on her qubit of the entangled pair $|r\rangle$, and sends her qubit to Bob.

	Transformation	New state
0	$(I \times I)\|r\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$
1	$(X \times I)\|r\rangle$	$\frac{1}{\sqrt{2}}(\|10\rangle + \|01\rangle)$
2	$(Z \times I)\|r\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$
3	$(Y \times I)\|r\rangle$	$\frac{1}{\sqrt{2}}(-\|10\rangle + \|01\rangle)$

Superdense coding

Bob

decodes the information applying a *CNOT* gate to the two qubits of the entangled pair and then H to the first qubit:

$$CNOT \longrightarrow \begin{bmatrix} \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \\ \frac{1}{\sqrt{2}}(|11\rangle + |01\rangle) \\ \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) \\ \frac{1}{\sqrt{2}}(-|11\rangle + |01\rangle) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \\ \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle) \otimes |1\rangle \\ \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle \\ \frac{1}{\sqrt{2}}(-|1\rangle + |0\rangle) \otimes |1\rangle \end{bmatrix}$$

$$H \otimes I \longrightarrow \begin{bmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{bmatrix}$$

Bob then measures the two qubits in the standard basis to obtain the 2-bit binary encoding of the number Alice wished to send

Quantum algorithms

Recall the generic idea: engineering quantum effects as computational resources

Classes of algorithms

- Algorithms with superpolynomial speed-up, typically based on the quantum Fourier transform, include Shor's algorithm for prime factorization. The level of resources (qubits) required is not yet currently available.
- Algorithms with quadratic speed-up, typically based on amplitude amplification, as in the variants of Grover's algorithm for unstructured search. Easier to implement in current NISQ technology, typical component of other algorithms.
- Quantum simulation — not covered in this course.

... and we are done!

What have we covered

- Reactive systems:
  - classical interaction (communication)
  - + programmed parallelism (operators, e.g. $\|$)
  - + engineering
- Quantum systems:
  - quantum interaction (entanglement)
  - + physical/natural parallelism (superposition)
  - + engineering

... and we are done!

Where to look further

- Reactive computation is the base of the everyware — namely in its extensions to hybrid (discrete-continuous) programming and cyber-physical systems.
  Covered in the Formal Methods profile in the MSc on Informatics Engineering.
- Quantum computation is an extremely young and challenging area, looking for young people either with a theoretic or experimental profile.
  Get in touch if you are interested in pursuing studies/research in the area at UMinho, INESC TEC and INL.

Universidade do Minho

HASLab HIGH-ASSURANCE SOFTWARE LABORATORY

INL INTERNATIONAL IBERIAN NANOTECHNOLOGY LABORATORY