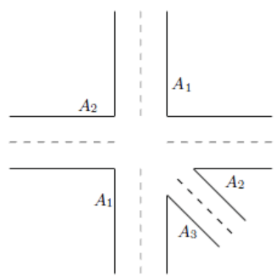
Interaction and Concurrency

Module 1: Reactive Systems
Problems 1 and 2

Until next 22 April 2024, please provide a complete, individual answer and quote suitably any reference (paper, book, software) used.

Problem 1

Consider the following junction where traffic is controlled by three traffic lights (processes A_1, A_2 and A_1):



1. The traffic controller
$$T \ \widehat{=}\ C \mid A_1 \mid A_2 \mid A_3$$
consists of three copies A_1, A_2 and A_3 of a process traffic light L, in parallel with a control process C. Process L enforces the usual infinite loop behaviour of a traffic light showing green, followed by yellow and then red, in cycle. Process C ensures that the green light is activated first in A_1, then in A_2 and finally in A_3, in a loop, avoiding any clashes. Specify processes L and C.
2. Use MCRL2 to draw the transition system of process T. What more can you do with MCRL2 to analyse the behaviour of process T?
3. Apply once the expansion theorem to process T. Comment the result you obtained.

Problem 2

Recall the *observable transition* relation $\stackrel{x}{\Longrightarrow} \subseteq \mathbb{P} \times \mathbb{P}$, where $x \in L = (Act - \{\tau\}) \cup \{\epsilon\}$. As discussed in the lectures, a $\stackrel{\epsilon}{\Longrightarrow}$-transition corresponds to zero or more transitions through an unobservable action τ.

Consider two new modal operators that express, respectively, the *possibility* and the *need* for a property to be valid after performing an arbitrary amount of unobservable behaviour.

$$
\begin{aligned}
E \models \langle\!\langle\rangle\!\rangle \phi \quad &\text{iff} \quad \exists_{F \in \{E' \mid E \stackrel{\epsilon}{\Longrightarrow} E'\}} \,.\, F \models \phi \\
E \models [\![\,]\!] \phi \quad &\text{iff} \quad \forall_{F \in \{E' \mid E \stackrel{\epsilon}{\Longrightarrow} E'\}} \,.\, F \models \phi
\end{aligned}
$$

By abbreviation we can now define the "observable versions " of $\langle K \rangle$ e $[K]$, for $K \subseteq L$. Thus,

$$
\begin{aligned}
\langle\!\langle K \rangle\!\rangle \phi &\stackrel{\text{abv}}{=} \langle\!\langle\rangle\!\rangle \; \langle K \rangle \; \langle\!\langle\rangle\!\rangle \phi \\
[\![K]\!] \phi &\stackrel{\text{abv}}{=} [\![\,]\!] \; [K] \; [\![\,]\!] \phi
\end{aligned}
$$

1. Explain the meaning of formulas $\langle\!\langle fdee \rangle\!\rangle \mathsf{true}$ and $[\![-]\!] \mathsf{false}$. Illustrate their use through the specification of four different, non-bisimilar processes such that $\langle\!\langle fdee \rangle\!\rangle \mathsf{true}$ holds in two of them and $[\![-]\!] \mathsf{false}$ in the other two.

2. In the logic you have studied in the lectures, formula

   $$\langle - \rangle \mathsf{true} \wedge [-a] \mathsf{false}$$

   expresses *inevitability*, i.e. the occurrence of action a is inevitable. Which of the formulas

   (a) $\langle\!\langle - \rangle\!\rangle \mathsf{true} \wedge [\![-a]\!] \mathsf{false}$

   (b) $[\![\,]\!] \; \langle\!\langle - \rangle\!\rangle \mathsf{true} \wedge [\![-a]\!] \mathsf{false}$

   if any, would express a similar property in the observational setting? Justify your answer. If none seems suitable, provide an alternative specification.

3. In the lectures, you have studied a close relationship between *bisimilarity* and *modal equivalence* for the logic then introduced. Discuss in some detail if and how a similar result holds relating *observational equivalence* and *modal equivalence* for the extended logic.